

**Anisotropic suppression of quasiparticle weight in
two-dimensional electron system with partially flat Fermi surface:
two-loop renormalization-group analysis**

Jun-ichiro Kishine* and Kenji Yonemitsu

Institute for Molecular Science, Okazaki 444-8585, Japan

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Abstract

Two-loop renormalization-group analysis for a two-dimensional electron system with a partially flat Fermi surface has been carried out. We found that, *irrespective of pairing mechanism*, the quasiparticle weight is anisotropically suppressed due to logarithmically singular processes in the flat regions of the Fermi surface. When the energy scale decreases, the quasiparticle weight is the most strongly suppressed around the center of the flat region, which qualitatively agrees with the anisotropic pseudogap behavior suggested through the angle-resolved photoemission spectroscopy experiments for underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$.

*E-mail:kishine@ims.ac.jp

Non-Fermi-liquid behavior in the normal state of high T_c superconductors has provoked a great deal of controversy.¹ Especially, the pseudogap phenomena in the underdoped region has been one of the main points at issue. Recent angle-resolved photoemission spectroscopy (ARPES) experiments on underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ ^{2,3} have revealed evidence that on cooling the pseudogap determined through the leading edge of the ARPES spectra first opens up around the points $(\pm\pi, 0)$ and $(0, \pm\pi)$ in the Brillouin zone below some crossover temperature $T^* \sim 180\text{K}$ without any broken symmetry. Then, the Fermi surface is broken up into the disconnected arcs which shrink with decreasing temperature above the superconducting transition temperature $T_c \sim 85\text{K}$.

Self-consistent treatments of the d -wave pairing fluctuation above T_c showed that the Fermi surface is broken up into the patches around $(\pm\pi, 0)$ and $(0, \pm\pi)$.⁴ Resonating valence bond picture also predicts the patchy Fermi surface due to spinon pairing.⁵ Here we propose another mechanism.

One aspect to study interacting fermion systems is to see how physical quantities *flow* toward their low-energy asymptotic behavior by means of the renormalization-group (RG) framework.⁶ Since the ARPES experiments detect the coherent one-particle excitations, the key issue here is the quasiparticle weight, $z(\mathbf{k})$, of an electron with a momentum \mathbf{k} . To see the RG flow of the quasiparticle weight, we have to carry out the RG analysis at least up to a “two-loop” order, since the lowest order perturbative correction to the quasiparticle weight comes from the logarithmically singular two-loop self energy diagram.⁷ In this work, by applying the two-loop RG analysis to a two-dimensional (2D) interacting electron system with a partially flat Fermi surface, we present a new scenario, *irrespective of pairing mechanism*, that the quasiparticle weight is anisotropically suppressed due to logarithmically singular processes in the flat regions of the Fermi surface.

We consider a 2D electron system with the model Fermi surface shown in Fig. 1 which resembles the real Fermi surface of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ ^{3,8} and consists of flat regions $(\alpha, \bar{\alpha}, \beta, \bar{\beta})$ with length 2Λ and round-arc regions (A, \bar{A}, B, \bar{B}) . The centers of the flat regions correspond to $(\pm\pi, 0)$ and $(0, \pm\pi)$ points in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$. The same model Fermi surface has been studied within the one-loop RG analysis (parquet approach) by Zheleznyak, Yakovenko and Dzyaloshinskii.⁹

Following the standard RG technique,⁶ we consider the linearized one-particle dispersion around the flat regions of the Fermi surface with bandwidth cutoff E_0 . We assume $E_0 \ll v_F\Lambda$ with v_F being the corresponding Fermi velocity. As in the case of one-dimensional (1D) metals,⁷ the electron one-particle and two-particle processes within the parallel flat regions give rise to the logarithmically singular contribution, $\ln[E_0/\omega]$ with $\omega (\leq E_0)$ being an infrared cutoff energy scale, to the two-particle scattering strengths and the quasiparticle

weight. On the other hand, the processes involving at least one electron in the round-arc regions and the processes involving two electrons in two flat regions perpendicular to each other, for example α and β , never give rise to this type of singular contribution. Consequently, for an electron located in the round-arc regions, the quasiparticle weight $z(\mathbf{k})$ remains marginal. Thus only the processes which are within the parallel flat regions are relevant to the RG flow of $z(\mathbf{k})$.⁹

We start with the action for the electron one-particle and two-particle processes in the parallel flat regions, α and $\bar{\alpha}$, $S_l = S_{\text{kin};l} + S_{\text{int};l}$, where l is a scaling parameter defined below. The kinetic action is given by

$$S_{\text{kin};l} = \sum_{\nu=\alpha,\bar{\alpha}} \sum_{\sigma} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} \int_{\mathcal{C}_{\nu;l}} \frac{d\mathbf{k}}{(2\pi)^2} \mathcal{G}_{\nu}^{-1}(k_x, i\varepsilon) c_{\nu\sigma}^*(K) c_{\nu\sigma}(K), \quad (1)$$

where $c_{\nu\sigma}^*$ and $c_{\nu\sigma}$ are the Grassmann variables representing the electron with the spin σ in the flat region $\nu = \alpha$ or $\bar{\alpha}$ and $K = (\mathbf{k}, \varepsilon)$ with ε being a Fermion thermal frequency. The non-interacting one-particle propagator is given by $\mathcal{G}_{\nu}^{-1}(k_x, i\varepsilon) = i\varepsilon - \varepsilon_{\nu}(k_x)$ where the linearized one-particle dispersions are given by $\varepsilon_{\alpha}(k_x) = v_F(k_x - k_F)$ and $\varepsilon_{\bar{\alpha}}(k_x) = v_F(-k_x - k_F)$ with v_F and k_F denoting the corresponding Fermi velocity and the Fermi momentum, respectively. We neglect the dependence of the Fermi velocity on k_y which is irrelevant in the RG sense. We parametrize the infrared cutoff energy as $\omega_l = E_0 e^{-l}$ with a scaling parameter, l . Two-dimensional electron momenta, \mathbf{k} , are restricted to the set $\mathcal{C}_{\nu;l} \equiv \{\mathbf{k} \mid \varepsilon_{\nu}(k_x) \leq \omega_l/2, -\Lambda \leq k_y \leq \Lambda\}$.

The action for the two-particle processes are written as

$$S_{\text{int};l} = \pi v_F \sum_{\sigma_1, \dots, \sigma_4} \prod_{i=1,2,3} \int_{-\infty}^{\infty} \frac{d\varepsilon_i}{2\pi} \int_{\mathcal{C}_l} \frac{d\mathbf{k}_i}{(2\pi)^2} g_l^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(k_{y_1}, k_{y_2}, k_{y_3}) c_{\bar{\alpha}\sigma_4}^*(K_4) c_{\alpha\sigma_3}^*(K_3) c_{\alpha\sigma_2}(K_2) c_{\bar{\alpha}\sigma_1}(K_1), \quad (2)$$

where $\prod_{i=1,2,3} \int_{-\infty}^{\infty} \frac{d\varepsilon_i}{2\pi} \int_{\mathcal{C}_l} \frac{d\mathbf{k}_i}{(2\pi)^2} = \int_{-\infty}^{\infty} \frac{d\varepsilon_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\varepsilon_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\varepsilon_3}{2\pi} \int_{\mathcal{C}_{\bar{\alpha};l}} \frac{d\mathbf{k}_1}{(2\pi)^2} \int_{\mathcal{C}_{\alpha;l}} \frac{d\mathbf{k}_2}{(2\pi)^2} \int_{\mathcal{C}_{\alpha;l}} \frac{d\mathbf{k}_3}{(2\pi)^2} \theta(\omega_l/2 - |\varepsilon_{\bar{\alpha}}(k_{x4})|) \theta(\Lambda - |k_{y4}|)$ with $\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3$. The two-particle scattering processes considered here are depicted in Fig. 2. Each leg of the scattering vertex is labeled by the regional index $\nu = \alpha, \bar{\alpha}$, the momentum in the k_y direction and the spin. The dimensionless scattering strengths are given by

$$g_l^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(k_{y_1}, k_{y_2}, k_{y_3}) = \delta_{\sigma_4 \sigma_2} \delta_{\sigma_3 \sigma_1} g_l^{(1)}(k_{y_1}, k_{y_2}, k_{y_3}) - \delta_{\sigma_4 \sigma_1} \delta_{\sigma_2 \sigma_3} g_l^{(2)}(k_{y_1}, k_{y_2}, k_{y_3}) \quad (3)$$

where $g_l^{(1)}(k_{y_1}, k_{y_2}, k_{y_3})$ and $g_l^{(2)}(k_{y_1}, k_{y_2}, k_{y_3})$ denote the strengths of the backward and forward scattering between the parallel flat regions, respectively. Within the present model, the electron umklapp process occurs in the special case where the distance between the parallel

flat regions is equal to π . In this work, we do not take account of this case and neglect the electron umklapp process.

We split up the set of \mathbf{k} -points, $\mathcal{C}_{\nu;l}$, into two subsets as $\mathcal{C}_{\nu;l} = \mathcal{C}_{\nu;l+dl}^< \oplus d\mathcal{C}_{\nu;l+dl}^>$, where $\mathcal{C}_{\nu;l+dl}^< \equiv \{\mathbf{k} \mid |\varepsilon_{\nu}(k_x)| \leq \omega_{l+dl}/2, -\Lambda \leq k_y \leq \Lambda\}$ (low-energy shell) and $d\mathcal{C}_{\nu;l+dl}^> \equiv \{\mathbf{k} \mid \omega_{l+dl}/2 \leq |\varepsilon_{\nu}(k_x)| \leq \omega_l/2, -\Lambda \leq k_y \leq \Lambda\}$ (high-energy shell). Accordingly, the action is decomposed as $S_l = S_{l+dl}^< + S_{l+dl}^>$. Integration over the modes in the high-energy shell gives the corresponding counterpart of the partition function as

$$Z = \int_{\mathcal{C}_{l+dl}^<} \mathcal{D}c^* \mathcal{D}c \exp \left[S_{l+dl}^< + \sum_{n=1}^{\infty} \frac{1}{n!} \langle\langle [S_{\text{int};l+dl}^>]^n \rangle\rangle_c \right], \quad (4)$$

where $\mathcal{D}c^* \mathcal{D}c$ symbolizes the measure over the Fermion Grassmann variables and $\mathcal{C}_{l+dl}^<$ means that Fermion momenta are restricted to the low-energy shell $\mathcal{C}_{\alpha;l+dl}^<$ or $\mathcal{C}_{\bar{\alpha};l+dl}^<$. The average over the modes in the high-energy shell is defined as $\langle\langle (\cdots) \rangle\rangle = Z_{>}^{-1} \int_{d\mathcal{C}_{l+dl}^>} \mathcal{D}c^* \mathcal{D}c \exp[S_{\text{kin};l+dl}^>] (\cdots)$, with $Z_{>} = \int_{d\mathcal{C}_{l+dl}^>} \mathcal{D}c^* \mathcal{D}c \exp[S_{\text{kin};l+dl}^>]$ and the subscript 'c' represents the connected diagrams. We perform a perturbative expansion of $\sum_{n=1}^{\infty} \frac{1}{n!} \langle\langle [S_{\text{int};l+dl}^>]^n \rangle\rangle_c$ by picking up the Feynmann diagrams whose contribution is in proportion to dl and then replacing $S_{l+dl}^< + \sum_{n=1}^{\infty} \frac{1}{n!} \langle\langle [S_{\text{int};l+dl}^>]^n \rangle\rangle_c$ with the renormalized action generally written in a form

$$\begin{aligned} \tilde{S}_{l+dl}^< &= \sum_{\nu=\alpha,\bar{\alpha}} \sum_{\sigma} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} \int_{\mathcal{C}_{\nu;l+dl}^<} \frac{d\mathbf{k}}{(2\pi)^2} [1 + \theta_l(k_y)dl + \mathcal{O}(dl^2)] \mathcal{G}_{\nu}^{-1}(k_x, i\varepsilon) c_{\nu\sigma}^*(K) c_{\nu\sigma}(K) \\ &+ \pi v_F \sum_{\sigma_1, \dots, \sigma_4} \prod_{i=1,2,3} \int_{-\infty}^{\infty} \frac{d\varepsilon_i}{2\pi} \int_{\mathcal{C}_{l+dl}^<} \frac{d\mathbf{k}_i}{(2\pi)^2} \left[g_l^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(k_{y1}, k_{y2}, k_{y3}) \right. \\ &\left. + w_l^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(k_{y1}, k_{y2}, k_{y3})dl + \mathcal{O}(dl^2) \right] c_{\bar{\alpha}\sigma_4}^*(K_4) c_{\alpha\sigma_3}^*(K_3) c_{\alpha\sigma_2}(K_2) c_{\bar{\alpha}\sigma_1}(K_1). \end{aligned} \quad (5)$$

The contribution $\theta_l(k_y) \mathcal{G}_{\nu}^{-1}(k_x, i\varepsilon)dl$ in the renormalized kinetic action comes from the two-loop self energy diagram depicted in Fig. 3, where $\theta_l(k_y)$ is given by

$$\theta_l(k_y) = \frac{1}{8} \left[2g_l^{(1)} \square g_l^{(1)} + 2g_l^{(2)} \square g_l^{(2)} - g_l^{(1)} \square g_l^{(2)} - g_l^{(2)} \square g_l^{(1)} \right], \quad (6)$$

where the operation represented by \square is defined as

$$g_l^{(i)} \square g_l^{(j)} = \int_{-\Lambda+k_y}^{\Lambda+k_y} \frac{dq_y}{2\pi} \int_{-\Lambda-\text{Min}(0,q_y)}^{\Lambda-\text{Max}(0,q_y)} \frac{dk_y'}{2\pi} g_l^{(i)}(k_y', k_y, k_y - q_y) g_l^{(j)}(k_y' + q_y, k_y - q_y, k_y). \quad (7)$$

The integration ranges are determined through the condition that all the incoming and outgoing momenta at each scattering vertex in Fig. 3 must be within the parallel flat regions. In the renormalized action for the two-particle scattering processes, the contribution $w_l^{(i)}(k_{y1}, k_{y2}, k_{y3})dl$, where $w_l^{\sigma_1 \sigma_2 \sigma_3 \sigma_4}(k_{y1}, k_{y2}, k_{y3}) = \delta_{\sigma_4 \sigma_2} \delta_{\sigma_3 \sigma_1} w_l^{(1)}(k_{y1}, k_{y2}, k_{y3}) - \delta_{\sigma_4 \sigma_1} \delta_{\sigma_2 \sigma_3} w_l^{(2)}(k_{y1}, k_{y2}, k_{y3})$, comes from the two-loop vertex correction diagrams depicted in Fig. 4. As space is limited, we do not write down full expression for $w_l^{(i)}(k_{y1}, k_{y2}, k_{y3})$.

To restore the original cutoff, we rescale the momenta and frequencies as $\tilde{\mathbf{k}} = (e^{dl}k_x, k_y) = (\tilde{k}_x, k_y)$ and $\tilde{\varepsilon} = e^{dl}\varepsilon$, respectively. Then, to leave the kinetic action scale-invariant, we simultaneously perform the scale transformation of the Fermion Grassmann variable as $\tilde{c}_{\nu\sigma}(\tilde{K}) = [1 + \frac{1}{2}\{\theta_l(k_y) - 3\}dl]c_{\nu\sigma}(K)$, where $\tilde{K} = (\tilde{k}_x, k_y, i\tilde{\varepsilon})$. Thus the two-loop RG equations for the scattering strengths are given by

$$\frac{dg_l^{(i)}(k_{y1}, k_{y2}, k_{y3})}{dl} = w_l^{(i)}(k_{y1}, k_{y2}, k_{y3}) - \frac{1}{2}g_l^{(i)}(k_{y1}, k_{y2}, k_{y3}) \sum_{j=1, \dots, 4} \theta_l(k_{y_j}), \quad (8)$$

where $i = 1, 2$ and $k_{y4} = k_{y1} + k_{y2} - k_{y3}$. The one-loop counterparts of the RG eqs. (8) are given by Zheleznyak, Yakovenko and Dzyaloshinskii,⁹ where $w_l^{(i)}(k_{y1}, k_{y2}, k_{y3})$ includes only one-loop vertex correction diagrams, Fig. 4(a) and 4(b), and the second term of the r.h.s of the RG eq. (8) does not appear. As an initial condition at $l = 0$, we put the scattering strengths equal to the momentum-independent Hubbard repulsion, U , i.e.

$$g_0^{(i)}(k_{y1}, k_{y2}, k_{y3}) = U/\pi v_F. \quad (9)$$

The RG equation for the quasiparticle weight is obtained through the renormalized kinetic action in eq. (5) and is written as

$$\frac{d \ln z_l(k_y)}{dl} = -\theta_l(k_y). \quad (10)$$

As an initial condition at $l = 0$, we put the quasiparticle weight equal to the non-interacting value, i.e.

$$z_0(k_y) = 1. \quad (11)$$

It is here instructive to see the k_y dependence of $z_l(k_y)$ by assuming that $g_l^{(i)}(k_{y1}, k_{y2}, k_{y3})$ remain at the initial value, $U/\pi v_F$. This assumption applies to the early-stage (i.e, high-energy) behavior of the RG flow. Then, eqs. (6), (7) and (10) give the result, $z_l(k_y) = [\omega_l/E_0]^{\tilde{U}^2(3\Lambda^2 - k_y^2)/16\pi^2}$ with $\tilde{U} = U/\pi v_F$, which indicates that the quasiparticle weight at the center of the flat region ($k_y = 0$) is the most strongly suppressed due solely to *the kinematical restriction to the logarithmically divergent processes*. In reality, however, we cannot obtain the 2-loop RG flow of $z_l(k_y)$ in a consistent manner, until we take account of *the anisotropic RG flow of the scattering strengths*.

We solve numerically the RG equations, (8) and (10), under the initial conditions, (9) and (11). Absolute value of the Hubbard repulsion, U , is not essential for the RG flow of the scattering strengths and the quasiparticle weight. From now on, we set the Hubbard repulsion at $U = 4.5\pi v_F$. Linear differential equations, (8) and (10), are treated as recursion equations with an infinitesimal difference step $dl = 0.08$. The numerical procedure at the

n -th step with the corresponding scaling parameter $l_n = ndl$ consists of three consecutive steps. First, we insert $g_{l_n}^{(i)}(k_{y1}, k_{y2}, k_{y3})$ into the expressions of $\theta_{l_n}(k_y)$ and $w_{l_n}^{(i)}(k_{y1}, k_{y2}, k_{y3})$. Second, we perform the integration over the momenta in the k_y direction by dividing the interval, $-\Lambda \leq k_y \leq \Lambda$, into 32 discrete points to obtain $\theta_{l_n}(k_y)$ and $w_{l_n}^{(i)}(k_{y1}, k_{y2}, k_{y3})$. Finally, we obtain $g_{l_{n+1}}^{(i)}(k_{y1}, k_{y2}, k_{y3})$ by using eq. (8).

In Fig. 5(a), we show the RG flow of the quasiparticle weight, $z_l(k_y)$, for $0 \leq l \leq 3.2$. When the energy scale decreases, the quasiparticle weight is the most strongly suppressed around the center of the flat region, $k_y = 0$, forming a dip there. In Fig. 5(b), we show the same RG flow for $3.2 \leq l \leq 4.08$. The k_y dependence of the quasiparticle weight undergoes a crossover behavior toward its low-energy asymptotics. There is some crossover scaling parameter $l_{\text{cross}} \sim 3$ around which the dip of $z_l(k_y)$ around $k_y = 0$ becomes flatter. Then, in the low-energy regime, $l > l_{\text{cross}}$, the region around which $z_l(k_y)$ is the most strongly suppressed moves from the center ($k_y = 0$) toward the edges ($k_y = \pm\Lambda$) and finally $z_l(k_y)$ approaches zero everywhere for $-\Lambda \leq k_y \leq \Lambda$ in the low-energy limit. The k_y -dependence of the quasiparticle weight in the high-energy regime corresponds to the anisotropic pseudogap behavior in underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ suggested by the ARPES experiments.^{2,3} In the present analysis, the crossover value of the scaling parameter, l_{cross} , corresponds to the temperature scale, $T_{\text{cross}} \sim 0.05E_0$. Thus we may expect that the temperature range covered by the ARPES experiments on the normal state of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ lies within the high-energy regime introduced here. We here again stress that, in the present scheme, the anisotropic RG flow of the quasiparticle weight occurs due solely to the logarithmically singular processes coming from the flat regions of the Fermi surface.

To see the reason why the crossover behavior of $z_l(k_y)$ occurs, in Fig. 6, we show the RG flow of the backward scattering strength, $g_l^{(1)}(k_{y1}, k_{y2}, k_{y3})$ at points, $(k_{y1}, k_{y2}, k_{y3}) = (0, 0, 0)$, $(\Lambda, -\Lambda, \Lambda)$, $(\Lambda, -\Lambda, -\Lambda)$, $(\Lambda, \Lambda, \Lambda)$. As an example, let us see the RG flow of $g_l^{(1)}(0, 0, 0)$. The monotonic decrease of $g_l^{(1)}(0, 0, 0)$ at the early stage of the renormalization reminds us of the RG flow of the backward scattering strength in the purely one-dimensional metal, where the backward scattering strength monotonously decreases and vanishes toward the low-energy asymptotics corresponding to the Tomonaga-Luttinger fixed point.⁷ In the present case in contrast to this, the RG flow of $g_l^{(1)}(0, 0, 0)$ has a very shallow minimum around $l \sim l^*$ and begins to increase for $l > l_{\text{cross}}$. As is seen in the inset in Fig. 6, the flow becomes nearly constant for $l > l_c \sim 4.1$ in the low-energy limit due to the two-loop vertex corrections, while, in the case of the one-loop RG analysis, the corresponding flow diverges around $l \sim l_c$.⁹

The crossover behavior of $g_l^{(1)}(0, 0, 0)$ originates from breakdown of the cancellation between the particle-particle (Cooper) loop[Fig. 4(a)] and the particle-hole (Peierls) loop[Fig. 4(b)]. As the renormalization process goes on, the breakdown becomes more and

more conspicuous and the Peierls loop tends to dominate the Cooper loop. Consequently, the spin-density-wave fluctuation becomes the most dominant fluctuation in the low-energy limit, provided that the regions α and $\bar{\alpha}$ are perfectly flat.⁹ The crossover behavior of the RG flow of $z_l(k_y)$ corresponds to the crossover behavior of the scattering strengths from the high-energy regime to the low-energy regime.

We here consider the effects of small but finite curvature of the Fermi surface in the regions α and $\bar{\alpha}$ which always exists in reality. In this case, at the energy scale smaller than the energy resolution, ΔE , which detects the Fermi surface curvature, the two-loop self-energy diagram never gives rise to the contribution of the type $\theta_l(k_y)\mathcal{G}_\nu^{-1}(k_x, i\varepsilon)dl$ in the renormalized kinetic action in eq.(5). Consequently, *the RG flow of $z_l(k_y)$ stops* around the scaling parameter, l_{curv} , defined as $\Delta E \sim E_0 e^{-l_{\text{curv}}}$. In Fig. 7, we take $l_{\text{curv}} = 3$ (corresponding to $\Delta E/E_0 \sim 0.05$) as an example and show the RG flows of $z_l(k_y)$ at some representative points on the Fermi surface. As the energy scale (or temperature scale) decreases, the quasiparticle weight at the point **a** on the round-arc region remains marginal, indicating that the round-arc regions of the Fermi surface are robust. In the flat regions, the quasiparticle weight at the center, **c**, is more strongly suppressed than that at the edge, **b**, but remains finite in the low-energy limit due to the curvature of the Fermi surface.

In our analysis, the RG flow of $z_l(k_y)$ becomes discontinuous at the edge points of the flat region (the point **b** in the inset of the Fig. 7), which seems unnatural. We give here a qualitative comment on the origin of this discontinuity. In the present analysis, we treated the flat regions as decoupled to the round-arc regions in the RG sense. However, strictly speaking, at finite energy scale, the flat regions and the round-arc regions can couple to each other through the scattering processes involving the very tiny region, the shaded area in Fig 8, which can be regarded as an extension of the flat region. Thus, if we take account of these processes, the discontinuity of $z_l(k_y)$ at the edge points will be removed. We do not inquire further into this point, since this superficial discontinuity is irrelevant to our qualitative understanding of the RG flow of $z_l(k_y)$ in the present model.

In summary, we have carried out the two-loop renormalization-group analysis for the two-dimensional electron system with the partially flat Fermi surface and found that the logarithmically singular processes coming from the flat regions cause the renormalization-group flow of the quasiparticle weight which depends on the location of the \mathbf{k} -point in the flat regions of the Fermi surface. When the energy scale decreases, the quasiparticle weight is the most strongly suppressed around the center of the flat region, forming a dip there which qualitatively agrees with the anisotropic pseudogap behavior suggested through the ARPES experiments.

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REFERENCES

- ¹ See, for example, M. Imada, A. Fujimori, and Y. Tokura, to appear in Rev. Mod. Phys. (1998).
- ² M. R. Norman, H. Ding, M. Randeria, J. C. Campuzano, T. Yokoya, T. Takeuchi, T. Takahashi, T. Mochiku, K. Kadowaki, P. Guptasarma, and D. G. Hinks, Nature **392**, 157 (1998).
- ³ H. Ding, T. Yokoya, J. C. Campuzano, T. Takahashi, M. Randeria, M. R. Norman, T. Mochiku, K. Kadowaki, and J. Giapintzakis, Nature **382**, 51 (1996); H. Ding, M. R. Norman, T. Yokoya, T. Takeuchi, M. Randeria, J. C. Campuzano, T. Takahashi, T. Mochiky, and K. Kadowaki, Phys. Rev. Lett. **78**, 2628 (1997).
- ⁴ J. R. Engelbrecht, A. Nazarenko, M. Randeria, and E. Dagotto, Phys. Rev. B **57**, 13406 (1998).
- ⁵ X. G. Wen and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996).
- ⁶ R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
- ⁷ J. Sólyom, Adv. Phys. **28**, 201 (1979).
- ⁸ D. S. Marshall *et al.*, Phys. Rev. Lett. **76**, 4841 (1996); A. G. Loeser *et al.*, Science **273**, 325 (1996).
- ⁹ A. T. Zheleznyak V. M. Yakovenko, and I. E. Dzyaloshinskii, Phys. Rev. B **55**, 3200 (1997).

FIGURES

FIG. 1. The model Fermi surface considered here.

FIG. 2. The two-particle scattering vertex considered here. The solid and broken lines represent the propagators for the electrons in the flat regions, α and $\bar{\alpha}$, respectively. The box represents the scattering vertex, $g_l^{\sigma_1\sigma_2\sigma_3\sigma_4}(k_{y1}, k_{y2}, k_{y3})$.

FIG. 3. The two-loop self-energy diagram.

FIG. 4. The diagrams which contribute, (a) to the one-loop vertex correction in the particle-particle (Cooper) channel, (b) to the one-loop vertex correction in the particle-hole (Peierls) channel, and (c) to the two-loop vertex corrections.

FIG. 5. k_y dependence of the renormalization-group flow of the quasiparticle weight, $z_l(k_y)$, (a) for $0 \leq l \leq 3.2$, and (b) for $3.2 \leq l \leq 4.08$.

FIG. 6. The renormalization-group flow of the backward scattering strength, $g_l^{(1)}(k_{y1}, k_{y2}, k_{y3})$ at $(k_{y1}, k_{y2}, k_{y3}) = (0, 0, 0)$, $(\Lambda, -\Lambda, \Lambda)$, $(\Lambda, -\Lambda, -\Lambda)$, $(\Lambda, \Lambda, \Lambda)$. In the inset, the same graph covering a wider range of the vertical axis is shown.

FIG. 7. The renormalization-group flow of the quasiparticle weight, $z_l(k_y)$, at some representative points on the Fermi surface.

FIG. 8. The shaded area also contributes to the renormalization-group flow of the quasiparticle weight, which is not taken into account in this study.

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